

PHYS 115 test notes

Fluids

Solids and liquids are 'condensed matter' – dense and incompressible.

$$\rho = \frac{m}{V} \quad p = \frac{\vec{F}}{\vec{A}} \quad (\text{area vector normal to surface}) \quad p_0 = 101.3 \text{ kPa} \quad p = p_0 + p_g$$

Geometry

$$\text{Sphere: } A = 4\pi r^2 \quad V = \frac{4}{3}\pi r^3$$

Vectors

$$|\vec{A} \cdot \vec{B}| = AB \cos \theta = |\vec{A}| |\vec{B}| \cos \theta = A_x B_x + A_y B_y + A_z B_z$$

(component of \vec{A} in direction of $\vec{B} \times |\vec{B}|$)

$$|\vec{A} \times \vec{B}| = AB \sin \theta \quad (\text{direction perpendicular to plane of } \vec{A} \text{ and } \vec{B})$$

θ : angle between A & B

$$A_x = A \cos \theta \quad A_y = A \sin \theta \quad \hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1 \quad \hat{i} \cdot \hat{j} = 0$$

Kinematics

Constant acceleration:

$$v = v_0 + at \quad v_{av} = \frac{1}{2}(v_0 + v) \quad x = x_0 + v_0 t + \frac{1}{2}at^2 \quad v^2 = v_0^2 + 2a \Delta x$$

$$\text{Projectiles: range } x = \frac{v_0^2 \sin(2\theta_0)}{g} \quad \text{flight time } t = \frac{2v_0 \sin \theta_0}{g}$$

$$\text{Uniform circular motion: } a_c = \frac{v^2}{r} = \omega^2 r$$

Circular motion with varying speed:

$$\text{radial / centripetal component } a_c = \frac{v^2}{r} \quad \text{tangential } a_t = \frac{dv}{dt}$$

Newton's laws

1. Isolated body at rest, no forces: remains at rest
Isolated body moving, no forces: constant velocity

2. $\vec{F} = m\vec{a}$ Constant force: constant acceleration

$$\vec{F} = \frac{d\vec{p}}{dt} \quad \vec{p} = m\vec{v} \quad \text{Conservation of momentum}$$

(Implies 1.)

3. Forces always act in pairs with opposite and equal magnitude.

Equivalence principle: Inertial mass and gravitational mass are equal. (Einstein)

$$\text{Gravity: } F_g = \frac{GMm}{r^2} \quad U = \frac{-GMm}{r}$$

$$\text{Rotational motion: } I = \sum_i m_i r_i^2 = \int r^2 dm \quad \vec{\tau} = \vec{r} \times \vec{F} \quad \tau = F r \sin \theta \quad \theta = \text{angle}$$

between force and radius

$$\text{Moment of inertia of uniform objects: Hoop: } MR^2 \quad \text{Cylinder: } \frac{1}{2}MR^2 \quad \text{Sphere: } \frac{2}{5}MR^2$$

$$\text{Rectangle: } \frac{1}{12}M(a^2 + b^2) \quad \text{Parallel axis theorem: } I = I_{CM} + MD^2$$

$$\text{Momentum: } \vec{\tau} = I\vec{\alpha} = \frac{d\vec{L}}{dt} \quad \vec{L} = I\vec{\omega} \quad \text{Linear density: } \lambda = \frac{dm}{dx}$$

$$\vec{\tau} = \vec{r} \times \vec{F} \quad \vec{v} = \vec{\omega} \times \vec{r} \quad \vec{L} = \vec{r} \times \vec{p} \quad \text{Air resistance - Stoke's law: } F_{\text{drag}} = 6\pi\eta r v$$

$$\text{Work: } W = \int \vec{F} \cdot d\vec{l}$$

Physical optics

$$\text{Intensity: } I = E^2 = \frac{P}{A} = \frac{P}{4\pi r^2} \quad \text{Radiation pressure: } P_{\text{rad}} = \frac{I}{c} \quad \text{Polarization: } I = I_0 \cos^2 \theta$$

$$\text{Phase shift through different media: } \Delta\phi = 2\pi \frac{L}{\lambda} (n_2 - n_1)$$

Interference

$m \in \mathbb{N}$, λ = wavelength in vacuum

Double slit / diffraction grating maxima: $d \sin \theta = m\lambda$ d = distance between slits

Single slit minima: $a \sin \theta = m\lambda$ a = slit width, $m \neq 0$

Circular aperture first minima: $a \sin \theta_1 = 1.22\lambda$ (also angular separation for resolvability)

$$\text{Thin film 123 constructive / 121 destructive interference: } \frac{2Ln_2}{\lambda} = m$$

Thermodynamics

$$\text{Thermal expansion: } \Delta L = \alpha L \Delta T \quad \Delta V = \beta V \Delta T \quad \beta \approx 3\alpha$$

Heat absorption by solids & liquids: $Q = C \Delta T = mc \Delta T$ C = heat capacity, c = specific heat

$$\text{Work done by gas: } W = \int_{V_i}^{V_f} p dV \quad \text{First law: } \Delta E_{\text{int}} = Q - W \quad \text{for all paths.}$$

$$\text{Conduction: } P_{\text{cond}} = \frac{Q}{t} = kA \frac{T_H - T_C}{L} \quad k = \text{thermal conductivity, } L = \text{conduction distance}$$

Radiation: $P_{\text{net}} = P_{\text{rad}} - P_{\text{abs}} = \epsilon \sigma A (T^4 - T_{\text{env}}^4)$ ϵ = emissivity ≤ 1 , $\sigma = 5.67 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$

Ideal gas law: $pV = nRT = NkT$ n = number of moles of gas, N = number of molecules of gas, R = universal gas constant = $8.31 \text{ Jmol}^{-1}\text{K}^{-1}$, k = Boltzmann's constant = $1.38 \times 10^{-23} \text{ JK}^{-1}$

Isothermal work: $W = nRT \ln\left(\frac{V_f}{V_i}\right)$

$v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}}$ m = mass of one molecule, M = molar mass

Translational kinetic energy: $K_{av} = \frac{3}{2}kT$ (polyatomic gas also has rotational)

$E_{int} = \frac{1}{2}fNkT = \frac{1}{2}fnRT$ f = 3 for monatomic gas

Molar specific heat at constant volume: $C_v = \frac{1}{2}fR$

Molar specific heat at constant pressure: $C_p = C_v + R$

Adiabatic expansion: $p_i V_i^\gamma = p_f V_f^\gamma$ $\gamma = \frac{C_p}{C_v}$

Entropy: same for all paths

For some reversible path i → f: $\Delta S = \int_f^i \frac{dQ}{T} = \frac{Q}{T}$ for isothermal processes

Second law: Entropy of a closed system increases for irreversible processes, stays constant (overall) for reversible.

Irreversible process where heat is transferred between different temp.s: $\Delta S = \frac{Q}{T_L} - \frac{Q}{T_H}$

Carnot engine: heat absorbed & discharged in isothermal processes. $W = Q_H + Q_L$

$\frac{|Q_H|}{T_H} = \frac{|Q_L|}{T_L}$ $Q_H > 0 > Q_L$

Efficiency: $\varepsilon = \frac{|W|}{|Q_H|} = 1 - \frac{T_L}{T_H}$ for Carnot engine **Third Law:** $T_L \neq 0K$

Electrostatics and magnetism

Coulomb's law: $F_{elec} = \frac{q_1 q_2}{4\pi \epsilon_0 r^2}$ $\epsilon_0 =$ permittivity of free space = $8.85 \times 10^{-12} C^2 N^{-1} m^{-2}$

$\vec{F} = q\vec{E}$

For a point charge q: $E = \frac{q}{4\pi \epsilon_0 r^2}$ Uniform wire: $E = \frac{\lambda}{2\pi \epsilon_0 r}$ Uniform charged plane:

$E = \frac{\sigma}{2\epsilon_0}$ Surface of conducting plane: $E = \frac{\sigma}{\epsilon_0}$ Ring, on axis: $E = \frac{Q}{4\pi \epsilon_0} \frac{x}{(R^2 + x^2)^{3/2}}$

For a dipole, on axis: $\vec{E} \approx \frac{1}{4\pi \epsilon_0} \frac{2\vec{p}}{z^3}$ perpendicular: $E \approx \frac{1}{4\pi \epsilon_0} \frac{p}{x^3}$ $x, z \gg d$

dipole moment: $\vec{p} = q\vec{d}$ directed from -ve to +ve

Torque on dipole in uniform electric field: $\vec{\tau} = \vec{p} \times \vec{E}$

Gauss's law: $\Phi = \oint_A \vec{E} \cdot d\vec{A} = \frac{\sum q}{\epsilon_0}$ Potential difference: $\Delta V = \frac{\Delta U}{q} = -\int \vec{E} \cdot d\vec{l}$

Work against an electric field: $W = \Delta U_{elec} = -q\vec{E} \cdot \vec{d} = -q \int \vec{E} \cdot d\vec{l}$

Capacitance: $C = \frac{\kappa \epsilon_0 A}{d}$ $Q = CV$ $W = \frac{1}{2}QV = \frac{1}{2}CV^2$ Charging:

$V_{cap} = \frac{q}{C} = V_B(1 - e^{-t/RC})$ $i = \frac{V_B}{R} e^{-t/RC}$ Discharging: $i = \frac{-V_B}{R} e^{-t/RC}$ $V_{cap} = V_B e^{-t/RC}$

Resistance: $\rho = \frac{m}{ne^2 \tau}$ $R = \frac{\rho L}{A}$ Ohm's law: $\vec{E} = \rho \vec{J}$ $\vec{J} = \frac{\vec{i}}{A}$

Magnetism: $\vec{F} = q\vec{v} \times \vec{B}$ $\vec{F} = L\vec{i} \times \vec{B}$ Bohr magnetism: $\mu_e = \frac{eh}{4\pi m_e}$ Moving conductor

in magnetic field: $\Delta V = \vec{v} \times \vec{B} \cdot \vec{L}$ Hall effect: $V_{hall} = \frac{iBd}{neA}$ Current loop: $\vec{\tau} = \vec{\mu} \times \vec{B}$

$\vec{\mu} = i\vec{A}$ Biot-Sewart: $\vec{B} = \frac{\mu_0 i}{4\pi} \int_{circuit} \frac{d\vec{l} \times \hat{r}}{r^2}$ $\mu_0 =$ permeability of free space = $4\pi \times 10^{-7} TmA^{-1}$

Field distance a from wire: $B = \frac{\mu_0 i (\cos \theta_1 - \cos \theta_2)}{4\pi a}$ Force between infinitely long current-

carrying wires: $\frac{F}{L} = \frac{\mu_0 i_1 i_2}{2\pi d}$ No monopoles: $\oint_{closed\ surface} \vec{B} \cdot d\vec{A} = 0$ Ampere's law:

$\oint_{closed\ path} \vec{B} \cdot d\vec{s} = \mu_0 \sum_{enc.} i + \mu_0 \epsilon_0 \frac{\partial \Phi_{elec}}{\partial t}$ Faraday's law: $\varepsilon = -\frac{\partial \Phi_{mag}}{\partial t}$ $\Phi_{mag} = \vec{B} \cdot \vec{A}$
 $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$

Trig. identities

Products

$2 \sin A \cos B = \sin(A+B) + \sin(A-B)$
 $2 \cos A \sin B = \sin(A+B) - \sin(A-B)$
 $2 \cos A \cos B = \cos(A+B) + \cos(A-B)$
 $2 \sin A \sin B = \cos(A-B) - \cos(A+B)$

Double angles

$\sin 2A = 2 \sin A \cos A$
 $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$
 $\cos 2A = \cos^2 A - \sin^2 A$

Compound angles

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
 $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
 $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$