PHYS 115 test notes

Fluids

Solids and liquids are 'condensed matter' – dense and incompressible.

$\rho = \frac{m}{V}$ $p = \frac{\vec{F}}{\vec{A}}$ (area vector normal to surface) $p_0 = 101.3\,\text{kPa}$ $p = p_0 + p_g$

Geometry

Sphere: $A = 4\pi r^2$ $V = \frac{4}{3}\pi r^3$

Vectors

$|\vec{A}\cdot\vec{B}| = AB\cos\theta = |\vec{A}||\vec{B}|\cos\theta = A_x B_x + A_y B_y + A_z B_z$

(component of $\vec{A}$ in direction of $\vec{B} \times |\vec{B}|$)

$|\vec{A}\times\vec{B}| = AB\sin\theta$ (direction perpendicular to plane of $\vec{A}$ and $\vec{B}$)

θ: angle between A & B

$A_x = A\cos\theta$ $A_y = A\sin\theta$ $\vec{\hat{\imath}}\cdot\vec{\hat{\imath}} = \vec{\hat{\jmath}}\cdot\vec{\hat{\jmath}} = \vec{\hat{k}}\cdot\vec{\hat{k}} = 1$ $\hat{\imath}\cdot\hat{\jmath} = 0$

Kinematics

Constant acceleration:

$v = v_0 + at$ $v_{\text{av}} = \frac{1}{2}(v_0 + v)$ $x = x_0 + v_0 t + \frac{1}{2}at^2$ $v^2 = v_0^2 + 2a\Delta x$

Projectiles: range $x = \frac{v_0^2 \sin(2\theta_0)}{g}$ flight time $t = \frac{2v_0\sin\theta_0}{g}$

Uniform circular motion: $a_c = \frac{v^2}{r} = \omega^2 r$

Circular motion with varying speed:

radial / centripetal component $a_c = \frac{v^2}{r}$ tangental $a_t = \frac{dv}{dt}$

Newton's laws

1. Isolated body at rest, no forces: remains at rest
   Isolated body moving, no forces: constant velocity
2. $\vec{F} = m\vec{a}$ Constant force: constant acceleration
   $\vec{F} = \frac{d\vec{p}}{dt}$ $\vec{p} = m\vec{v}$ Conservation of momentum
   (Implies 1.)
3. Forces always act in pairs with opposite and equal magnitude.

Equivalence principle: Inertial mass and gravitational mass are equal. (Einstein)

Gravity: $F_g = \frac{GMm}{r^2}$ $U = \frac{-G\,M\,m}{r}$

Rotational motion: $I = \sum_i m_i r_i^2 = \int r^2\,dm$ $\vec{\tau} = \vec{F}\times\vec{r}$ $\tau = F\,r\sin\theta$ θ = angle between force and radius

Moment of inertia of uniform objects: Hoop: $M\,R^2$ Cylinder: $\frac{1}{2}M\,R^2$ Sphere: $\frac{2}{5}M\,R^2$ Rectangle: $\frac{1}{12}M(a^2+b^2)$ **Parallel axis theorem:** $I = I_{CM} + M\,D^2$

Momentum: $\vec{\tau} = I\,\vec{\alpha} = \frac{d\vec{L}}{dt}$ $\vec{L} = I\,\vec{\omega}$ Linear density: $\lambda = \frac{dm}{dx}$

$\vec{\tau} = \vec{r}\times\vec{F}$ $\vec{v} = \vec{\omega}\times\vec{r}$ $\vec{L} = \vec{r}\times\vec{p}$ **Air resistance – Stoke's law:** $F_{\text{drag}} = 6\pi\eta r v$

Work: $W = \int \vec{F}\cdot d\vec{l}$

Physical optics

Intensity: $I = E^2 = \frac{P}{A} = \frac{P}{4\pi r^2}$ Radiation pressure: $P_{rad} = \frac{I}{c}$ Polarization: $I = I_0\cos^2\theta$

Phase shift through different media: $\Delta\phi = 2\pi\frac{L}{\lambda}(n_2 - n_1)$

Interference

m ∈ ℕ, λ = wavelength in vacuum

Double slit / diffraction grating maxima: $d\sin\theta = m\lambda$ d = distance between slits

Single slit minima: $a\sin\theta = m\lambda$ a = slit width, m ≠ 0

Circular aperture first minima: $a\sin\theta_1 = 1.22\lambda$ (also angular separation for resolvability)

Thin film 123 constructive / 121 destructive interference: $\frac{2Ln_2}{\lambda} = m$

Thermodynamics

Thermal expansion: $\Delta L = \alpha L\Delta T$ $\Delta V = \beta V\Delta T$ $\beta \approx 3\alpha$

Heat absorption by solids & liquids: $Q = C\Delta T = mc\Delta T$ C = heat capacity, c = specific heat

Work done by gas: $W = \int_{V_i}^{V_f} p\,dV$ First law: $\Delta E_{\text{int}} = Q - W$ for all paths.

Conduction: $P_{\text{cond}} = \frac{Q}{t} = kA\frac{T_H - T_C}{L}$ k = thermal conductivity, L = conduction distance

Radiation: $P_{\text{net}} = P_{\text{rad}} - P_{\text{abs}} = \varepsilon\sigma A(T^4 - T_{\text{env}}^4)$ ε = emissivity ≤ 1, σ = 5.67 * 10^{-8} $\text{Wm}^{-2}\text{K}^{-4}$

Ideal gas law: $pV = nRT = NkT$ n = number of moles of gas, N = number of molecules of gas, R = universal gas constant = 8.31 $\text{Jmol}^{-1}\text{K}^{-1}$, k = Boltzmann's constant = 1.38 * 10^{-23} JK^{-1}

Isothermal work: $W=nRT\ln\left(\frac{V_f}{V_i}\right)$

$v_{\text{rms}}=\sqrt{\frac{3kT}{m}}=\sqrt{\frac{3RT}{M}}$ m = mass of one molecule, M = molar mass

Translational kinetic energy: $K_{\text{av}}=\frac{3}{2}kT$ (polyatomic gas also has rotational)

$E_{\text{int}}=\frac{1}{2}fNkT=\frac{1}{2}fnRT$ f = 3 for monatomic gas

Molar specific heat at constant volume: $C_v=\frac{1}{2}fR$

Molar specific heat at constant pressure: $C_p=C_v+R$

Adiabatic expansion: $p_iV_i^{\gamma}=p_fV_f^{\gamma}$ $\gamma=\frac{C_p}{C_v}$

Entropy: same for all paths

For some reversible path i → f: $\Delta S=\int_f^i\frac{dQ}{T}$ $=\frac{Q}{T}$ for isothermal processes

Second law: Entropy of a closed system increases for irreversible processes, stays constant (overall) for reversible.

Irreversible process where heat is transferred between different temp.s: $\Delta S=\frac{Q}{T_L}-\frac{Q}{T_H}$

Carnot engine: heat absorbed & discharged in isothermal processes. $W=Q_H+Q_L$

$\frac{|Q_H|}{T_H}=\frac{|Q_L|}{T_L}$ $Q_H>0>Q_L$

Efficiency: $\varepsilon=\frac{|W|}{|Q_H|}$ $=1-\frac{T_L}{T_H}$ for Carnot engine **Third Law:** $T_L\neq 0\,\text{K}$

Electrostatics and magnetism

Coulomb's law: $F_{\text{elec}}=\frac{q_1q_2}{4\pi\varepsilon_0r^2}$ ε_0 = permittivity of free space = 8.85 * 10^{-12} $C^2N^{-1}m^{-2}$

$\vec{F}=q\vec{E}$

For a point charge q: $E=\frac{q}{4\pi\varepsilon_0r^2}$ Uniform wire: $E=\frac{\lambda}{2\pi\varepsilon_0r}$ Uniform charged plane:

$E=\frac{\sigma}{2\varepsilon_0}$ Surface of conducting plane: $E=\frac{\sigma}{\varepsilon_0}$ Ring, on axis: $E=\frac{Q}{4\pi\varepsilon_0}\frac{x}{(R^2+x^2)^{3/2}}$

For a dipole, on axis: $\vec{E}\approx\frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{z^3}$ perpendicular: $E\approx\frac{1}{4\pi\varepsilon_0}\frac{p}{x^3}$ $x,z\gg d$

dipole moment: $\vec{p}=q\vec{d}$ directed from -ve to +ve

Torque on dipole in uniform electric field: $\vec{\tau}=\vec{p}\times\vec{E}$

Gauss's law: $\Phi=\oint_A\vec{E}\cdot d\vec{A}=\frac{\sum q}{\varepsilon_0}$ Potential difference: $\Delta V=\frac{\Delta U}{q}=-\int\vec{E}\cdot d\vec{l}$

Work against an electric field: $W=\Delta U_{\text{elec}}=-q\vec{E}\cdot\vec{d}=-q\int\vec{E}\cdot d\vec{l}$

Capacitance: $C=\frac{\kappa\varepsilon_0A}{d}$ $Q=CV$ $W=\frac{1}{2}QV=\frac{1}{2}CV^2$ Charging:

$V_{\text{cap}}=\frac{q}{C}=V_{\text{B}}(1-e^{\frac{-t}{RC}})$ $i=\frac{V_{\text{B}}}{R}e^{\frac{-t}{RC}}$ Discharging: $i=\frac{-V_{\text{B}}}{R}e^{\frac{-t}{RC}}$ $V_{\text{cap}}=V_{\text{B}}e^{\frac{-t}{RC}}$

Resistance: $\rho=\frac{m}{ne^2\tau}$ $R=\frac{\rho L}{A}$ Ohm's law: $\vec{E}=\rho\vec{J}$ $\vec{J}=\frac{\vec{i}}{A}$

Magnetism: $\vec{F}=q\vec{v}\times\vec{B}$ $\vec{F}=L\vec{i}\times\vec{B}$ Bohr magnetism: $\mu_e=\frac{eh}{4\pi m_e}$ Moving conductor in magnetic field: $\Delta V=\vec{v}\times\vec{B}\cdot\vec{L}$ Hall effect: $V_{\text{hall}}=\frac{iB}{ne}\frac{d}{A}$ Current loop: $\vec{\tau}=\vec{\mu}\times\vec{B}$

$\vec{\mu}=i\vec{A}$ Biot-Sewart: $\vec{B}=\frac{\mu_0i}{4\pi}\int_{\text{circuit}}\frac{\vec{\delta l}\times\vec{\hat{r}}}{r^2}$ μ_0 = permeability of free space = $4\pi*10^{-7}$ TmA^{-1}

Field distance *a* from wire: $B=\frac{\mu_0i(\cos\theta_1-\cos\theta_2)}{4\pi a}$ Force between infinitely long current-carrying wires: $\frac{F}{L}=\frac{\mu_0i_1i_2}{2\pi d}$ No monopoles: $\oint_{\text{closed surface}}\vec{B}\cdot d\vec{A}=0$ Ampere's law:

$\oint_{\text{closed path}}\vec{B}\cdot d\vec{s}=\mu_o\sum_{\text{enc.}}i+\mu_0\varepsilon_0\frac{\partial\Phi_{\text{elec}}}{\partial t}$ Faraday's law: $\varepsilon=-\frac{\partial\Phi_{\text{mag}}}{\partial t}$ $\Phi_{\text{mag}}=\vec{B}\cdot\vec{A}$

$c=\frac{1}{\sqrt{\varepsilon_0\mu_0}}$

Trig. identities

Products

$$2\sin A\cos B=\sin(A+B)+\sin(A-B)$$
$$2\cos A\sin B=\sin(A+B)-\sin(A-B)$$
$$2\cos A\cos B=\cos(A+B)+\cos(A-B)$$
$$2\sin A\sin B=\cos(A-B)-\cos(A+B)$$

Double angles

$$\sin 2A=2\sin A\cos A$$
$$\tan 2A=\frac{2\tan A}{1-\tan^2A}$$
$$\cos 2A=\cos^2A-\sin^2A$$

Compound angles

$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$
$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$
$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$