

PHYS 114 test notes

Optics

Fermat's principle: ray passes from A to B in minimum time.

Ray bends towards normal as it enters a denser medium.

Snell's law: $n_1 \sin \theta_1 = n_2 \sin \theta_2$

$$d_o^{-1} + d_i^{-1} = f^{-1} \quad \frac{h_i}{h_o} = -\frac{d_i}{d_o} \quad m = \frac{h_i}{h_o}$$

$m < 0$: image inverted

$d_i < 0$: virtual image (and upright)

Camera:

f-number = f/D D is effective aperture

intensity $I \propto \text{f-number}^{-2}$

$$m = \frac{.25 \text{ m}}{f} \quad \text{For human eye, image at } \infty$$

Fluids

Solids and liquids are 'condensed matter' – dense and incompressible.

$$\rho = \frac{m}{V} \quad p = \frac{\vec{F}}{\vec{A}} \quad (\text{area vector normal to surface}) \quad p_0 = 101.3 \text{ kPa} \quad p = p_0 + p_g$$

Hydrostatics:

$$\Delta p = -\rho g \Delta y \quad p = p_0 + \rho g h \quad \text{Hydraulic press: } \frac{F_i}{A_i} = \frac{F_o}{A_o} \quad F_i d_i = F_o d_o$$

$$A_i d_i = A_o d_o \quad \text{Archimedes principle: } F_b = m_f g \quad m_f = \text{mass of fluid displaced}$$

Ideal fluid:

Steady flow (constant flow at given point, laminar, streamlines); incompressible (fixed density); inviscid

Fluid dynamics:

$$p_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2 \quad \text{Streamline rule: } A_1 v_1 = A_2 v_2$$

Kinematics

Constant acceleration:

$$v = v_0 + at \quad v_{\text{av}} = \frac{1}{2}(v_0 + v) \quad x = x_0 + v_0 t + \frac{1}{2} a t^2 \quad v^2 = v_0^2 + 2a \Delta x$$

Vectors:

$$|\vec{A} \cdot \vec{B}| = AB \cos \theta = |\vec{A}| |\vec{B}| \cos \theta \quad (\text{component of } \vec{A} \text{ in direction of } \vec{B} \times |\vec{B}|)$$

$$|\vec{A} \times \vec{B}| = AB \sin \theta \quad (\text{direction perpendicular to plane of } \vec{A} \text{ and } \vec{B})$$

θ : angle between A & B

$$A_x = A \cos \theta \quad A_y = A \sin \theta \quad \hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = 1 \quad \hat{i} \cdot \hat{j} = 0$$

Projectiles

$$\text{range } x = \frac{v_0^2 \sin(2\theta_0)}{g} \quad \text{flight time } t = \frac{2 v_0 \sin \theta_0}{g}$$

Uniform circular motion

$$a_c = \frac{v^2}{r}$$

Circular motion with varying speed

$$\text{radial / centripetal component } a_c = \frac{v^2}{r} \quad \text{tangential } a_t = \frac{dv}{dt}$$

Newton's laws

1. Isolated body at rest, no forces: remains at rest
Isolated body moving, no forces: constant velocity
2. $F = ma$ Constant force: constant acceleration

$$F = \frac{d\rho}{dt} \quad \rho = mv \quad \text{Conservation of momentum}$$

(Implies 1.)

3. Forces always act in pairs with opposite and equal magnitude.

Equivalence principle: Inertial mass and gravitational mass are equal. (Einstein)

Gravity

$$F_g = \frac{GMm}{r^2} \quad U = \frac{-GMm}{r}$$

Rotational motion:

$$I = \sum m r^2 = \int r^2 dm \quad \tau = \vec{F} \times \vec{r} = F r \sin \theta \quad \theta = \text{angle between force and radius}$$

Moment of inertia of uniform objects: Hoop: $M R^2$ Cylinder: $\frac{1}{2} M R^2$ Sphere:

$$\frac{2}{5} M R^2 \quad \text{Rectangle: } \frac{1}{12} M (a^2 + b^2)$$

Parallel axis theorem: $I = I_{CM} + M D^2$

Simple Harmonic Motion

$$x = x_m \cos(\omega t + \phi) \quad \omega = 2\pi f \quad v = -\omega x_m \sin(\omega t + \phi) \quad a = -\omega^2 x$$

$$F = -kx \quad k = m\omega^2 \quad K = \frac{1}{2} m v^2 \quad \text{Potential of spring: } U = \frac{1}{2} k x^2 \quad E = \frac{1}{2} k x_m^2$$

Angular SHM:

$$\tau = -\kappa \theta \quad \alpha = -\frac{\kappa}{I} \theta \quad ??? \quad K = \frac{1}{2} I \omega^2$$

Simple pendulum:

Approximating $\sin \theta = \theta$: $\tau \approx -mgL\theta \quad \omega = \sqrt{\frac{g}{L}}$

Physical pendulum:

$$\omega = \sqrt{\frac{mgh}{I_0}} \quad h = \text{distance from centre of mass to suspension point; } I_0 = \text{moment of inertia}$$

$l =$ length of simple pendulum with same period; point at distance l from suspension point is 'centre of percussion' or 'centre of oscillation'.

Damped SHM:

Damping force $F_d = -bv \quad x = x_m e^{-\frac{bt}{2m}} \cos(\omega' t + \phi) \quad \omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$

Weak damping: $b^2 \ll 4km \quad \omega' \approx \omega$

Critical damping: $b^2 = 4km \quad \omega' \approx 0 \quad ???$ (No oscillation, just decay)

Over damping: $b^2 > 4km$ (No oscillation, but slower decay)

Waves

$$y = y_m \sin(kx - \omega t) = y_m \sin\left(\frac{2\pi}{\lambda} x - \frac{2\pi}{T} t\right) \quad (\text{positive velocity}) \quad k = \text{angular}$$

wavenumber $v = \lambda f \quad v = \sqrt{\frac{\tau}{\mu}} \quad P_{av} = \frac{1}{2} \mu v \omega^2 y_m^2$ At a given time & position, $K = U$.

$$u = \frac{dy}{dt}$$

Standing waves:

$$\lambda = \frac{2L}{n} \quad \lambda = \frac{4L}{2n-1}$$

Sound in air:

$$v = \sqrt{\frac{B}{\rho}} \quad B = \text{bulk modulus} \quad B = \frac{\Delta p V}{\Delta V} \quad \text{For ideal gas: } B = \gamma R T \quad R = 8.314 \text{ J K}^{-1} \text{ mol}^{-1}$$

$$s = s_m \cos(kx - \omega t) \quad \Delta p = \Delta p_m \sin(kx - \omega t) \quad \Delta p_m = v \rho \omega s_m$$

$$I = \frac{P_s}{4\pi r^2} = \frac{P_{av}}{A} = \frac{1}{2} \rho v \omega^2 s_m^2 = \frac{\Delta p_m^2}{2v\rho} \quad \beta = (10 \text{ dB}) \log \frac{I}{10^{-12} \text{ W m}^{-2}}$$

Doppler effect:

$$f' = f \frac{v \pm v_d}{v \pm v_s} \quad f' = f \left(1 + \frac{v_d}{v}\right) \approx f \left(1 + \frac{v_s}{v}\right)$$

For electromagnetic radiation: $f' = f \sqrt{\frac{c+v}{c-v}}$ Mach cone: $\sin \theta = \frac{v}{v_s}$

Quantum

$$E = m_0 c^2 + K \quad E^2 = m_0^2 c^4 + p^2 c^2 \quad E = hf \quad \lambda = \frac{h}{p}$$

$$\Psi = A e^{i(kx - \omega t)} = \psi e^{-i\omega t} \quad |\Psi|^2 \text{ is probability density} \quad h = 6.63 \times 10^{-34} \text{ J s} \quad e = -1.60 \times 10^{-19} \text{ C}$$

Trig. identities

Products

$$2 \sin A \cos B = \sin(A+B) + \sin(A-B)$$

$$2 \cos A \sin B = \sin(A+B) - \sin(A-B)$$

$$2 \cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$2 \sin A \sin B = \cos(A-B) - \cos(A+B)$$

Compound angles

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

Sums

$$\sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$\sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2}$$

$$\cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$\cos C - \cos D = -2 \sin \frac{C+D}{2} \sin \frac{C-D}{2}$$

Double angles

$$\sin 2A = 2 \sin A \cos A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\cos 2A = \cos^2 A - \sin^2 A$$