PHYS 114 test notes

Optics

Fermat's principle: ray passes from A to B in minimum time.
Ray bends towards normal as it enters a denser medium.

Snell's law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

$$d_o^{-1} + d_i^{-1} = f^{-1} \qquad \frac{h_i}{h_o} = -\frac{d_i}{d_o} \qquad m = \frac{h_i}{h_o}$$

$m < 0$: image inverted
$d_i < 0$: virtual image (and upright)

Camera:

$\text{f-number} = f/D$ D is effective aperture

$\text{intensity } I \propto \text{f-number}^{-2}$

$m = \frac{.25\,\text{m}}{f}$ For human eye, image at ∞

Fluids

Solids and liquids are 'condensed matter' – dense and incompressible.

$\rho = \frac{m}{V}$ $\quad p = \frac{\vec{F}}{\vec{A}}$ (area vector normal to surface) $\quad p_0 = 101.3\,\text{kPa}$ $\quad p = p_0 + p_g$

Hydrostatics:

$\Delta p = -\rho g \Delta y$ $\quad p = p_0 + \rho g h$ **Hydraulic press:** $\frac{F_i}{A_i} = \frac{F_o}{A_o}$ $\quad F_i d_i = F_o d_o$

$A_i d_i = A_o d_o$ **Archimedes principle:** $F_b = m_f g$ $\quad m_f$ = mass of fluid displaced

Ideal fluid:

Steady flow (constant flow at given point, laminar, streamlines); incompressible (fixed density); inviscid

Fluid dynamics:

$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$ **Streamline rule:** $A_1 v_1 = A_2 v_2$

Kinematics

Constant acceleration:

$$v = v_0 + at \qquad v_{\text{av}} = \tfrac{1}{2}(v_0 + v) \qquad x = x_0 + v_0 t + \tfrac{1}{2} a t^2 \qquad v^2 = v_0^2 + 2a\Delta x$$

Vectors:

$|\vec{A}\cdot\vec{B}| = AB\cos\theta = |\vec{A}||\vec{B}|\cos\theta$ (component of $\vec{A}$ in direction of $\vec{B}$ × $|\vec{B}|$)

$|\vec{A}\times\vec{B}| = AB\sin\theta$ (direction perpendicular to plane of $\vec{A}$ and $\vec{B}$)

θ: angle between A & B

$A_x = A\cos\theta \qquad A_y = A\sin\theta \qquad \vec{\imath}\cdot\vec{\imath} = \vec{\jmath}\cdot\vec{\jmath} = 1 \qquad \hat{\imath}\cdot\hat{\jmath} = 0$

Projectiles

range $x = \frac{v_0^2 \sin(2\theta_0)}{g}$ $\quad$ flight time $t = \frac{2 v_0 \sin\theta_0}{g}$

Uniform circular motion

$$a_c = \frac{v^2}{r}$$

Circular motion with varying speed

radial / centripetal component $a_c = \frac{v^2}{r}$ $\quad$ tangental $a_t = \frac{dv}{dt}$

Newton's laws

1. Isolated body at rest, no forces: remains at rest
   Isolated body moving, no forces: constant velocity
2. $F = ma$ Constant force: constant acceleration
   $F = \frac{d\rho}{dt}$ $\quad \rho = mv$ Conservation of momentum
   (Implies 1.)
3. Forces always act in pairs with opposite and equal magnitude.

Equivalence principle: Inertial mass and gravitational mass are equal. (Einstein)

Gravity

$$F_g = \frac{GMm}{r^2} \qquad U = \frac{-GMm}{r}$$

Rotational motion:

$I = \sum m r^2 = \int r^2\, dm$ $\quad \tau = \vec{F}\times\vec{r} = F r \sin\theta$ θ = angle between force and radius

Moment of inertia of uniform objects: Hoop: MR^2 Cylinder: $\frac{1}{2}MR^2$ Sphere: $\frac{2}{5}MR^2$ Rectangle: $\frac{1}{12}M(a^2+b^2)$

Parallel axis theorem: $I = I_{CM} + MD^2$

Simple Harmonic Motion

$x = x_m \cos(\omega t + \phi)$ $\omega = 2\pi f$ $v = -\omega x_m \sin(\omega t + \phi)$ $a = -\omega^2 x$

$F = -kx$ $k = m\omega^2$ $K = \frac{1}{2} m v^2$ **Potential of spring:** $U = \frac{1}{2} k x^2$ $E = \frac{1}{2} k x_m^2$

Angular SHM:

$\tau = -\kappa \theta$ $\alpha = -\frac{\kappa}{I}\theta$??? $K = \frac{1}{2} I \omega^2$

Simple pendulum:

Approximating sin θ = θ: $\tau \approx -m g L \theta$ $\omega = \sqrt{\frac{g}{L}}$

Physical pendulum:

$\omega = \sqrt{\frac{m g h}{I_0}}$ h = distance from centre of mass to suspension point; I_0 = moment of inertia

l = length of simple pendulum with same period; point at distance l from suspension point is 'centre of percussion' or 'centre of oscillation'.

Damped SHM:

Damping force $F_d = -bv$ $x = x_m e^{-\frac{bt}{2m}} \cos(\omega' t + \phi)$ $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$

Weak damping: $b^2 \ll 4km$ $\omega' \approx \omega$

Critical damping: $b^2 = 4km$ $\omega' \approx 0$??? (No oscillation, just decay)

Over damping: $b^2 > 4km$ (No oscillation, but slower decay)

Waves

$y = y_m \sin(kx - \omega t) = y_m \sin(\frac{2\pi}{\lambda} x - \frac{2\pi}{T} t)$ (positive velocity) k = angular wavenumber $v = \lambda f$ $v = \sqrt{\frac{\tau}{\mu}}$ $P_{av} = \frac{1}{2} \mu v \omega^2 y_m^2$ At a given time & position, K = U.

$u = \frac{dy}{dt}$

Standing waves:

$\lambda = \frac{2L}{n}$ $\lambda = \frac{4L}{2n-1}$

Sound in air:

$v = \sqrt{\frac{B}{\rho}}$ B = bulk modulus $B = \frac{\Delta p V}{\Delta V}$ For ideal gas: $B = \gamma R T$ R=8.314 J $K^{-1}mol^{-1}$

$s = s_m \cos(kx - \omega t)$ $\Delta p = \Delta p_m \sin(kx - \omega t)$ $\Delta p_m = v \rho \omega s_m$

$I = \frac{p_s}{4\pi r^2} = \frac{p_{av}}{A} = \frac{1}{2} \rho v \omega^2 s_m^2 = \frac{\Delta p_m^2}{2 v \rho}$ $\beta = (10\,\mathrm{dB}) \log \frac{I}{10^{-12}\,\mathrm{W\,m^{-2}}}$

Doppler effect:

$f' = f \frac{v \pm v_d}{v \pm v_s}$ $f' = f(1 + \frac{v_d}{v}) \approx f(1 + \frac{v_s}{v})$

For electromagnetic radiation: $f' = f \sqrt{\frac{c+v}{c-v}}$ Mach cone: $\sin\theta = \frac{v}{v_s}$

Quantum

$E = m_0 c^2 + K$ $E^2 = m_0^2 c^4 + p^2 c^2$ $E = hf$ $\lambda = \frac{h}{p}$

$\Psi = A e^{i(kx - \omega t)} = \psi e^{-i\omega t}$ $|\Psi|^2$ is probability density h=6.63*10^{-34} J s e=-1.60*10^{-19} C

Trig. identities

Products

$2 \sin A \cos B = \sin(A+B) + \sin(A-B)$

$2 \cos A \sin B = \sin(A+B) - \sin(A-B)$

$2 \cos A \cos B = \cos(A+B) + \cos(A-B)$

$2 \sin A \sin B = \cos(A-B) - \cos(A+B)$

Compound angles

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

Sums

$\sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$

$\sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2}$

$\cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2}$

$\cos C - \cos D = -2 \sin \frac{C+D}{2} \sin \frac{C-D}{2}$

Double angles

$\sin 2A = 2 \sin A \cos A$

$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$

$\cos 2A = \cos^2 A - \sin^2 A$